

Neutrino masses with inverse hierarchy from broken $L_e - L_\mu - L_\tau$: a reappraisal

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ABSTRACT: We discuss a class of models of neutrino masses and mixings with inverse hierarchy based on a broken $U(1)_F$ flavour symmetry with charge $L_e - L_\mu - L_\tau$ for lepton doublets and arbitrary right-handed charges. The symmetry breaking sector receives separate contributions from flavon vev breaking terms and from soft mass breaking in the right handed Majorana sector. The model is able to reproduce in a natural way all observed features of the charged lepton mass spectrum and of neutrino masses and mixings (even with arbitrarily small θ_{13}), with the exception of a moderate fine tuning which is needed to accommodate the observed small value of $r = \Delta m_{sol}^2 / \Delta m_{atm}^2$.

KEYWORDS: Neutrino Physics, Beyond Standard Model.

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1. Introduction

As well known [1], a simple dynamical approach to construct models of neutrino masses and mixings with inverse hierarchy is to start in first approximation from a $U(1)_F$ flavour symmetry [2] with charge $L_e - L_\mu - L_\tau$ [3], where L_i are the separate lepton numbers with flavour i . If this charge is assigned to the 3 families of $SU(2)$ lepton doublets l , the neutrino mass matrix $m_\nu \sim l^T l$, in the limit of exact symmetry, is of the form

$$m_\nu = m \begin{pmatrix} 0 & 1 & x \\ 1 & 0 & 0 \\ x & 0 & 0 \end{pmatrix}, \tag{1.1}$$

The eigenvalues are $(m_1, m_2, m_3) \sim (m, -m, 0)$, which indeed correspond to an inverse hierarchy spectrum with $m^2 \sim \Delta m_{atm}^2$. The opposite sign of m_1 and m_2 makes the spectrum sufficiently stable under renormalisation group corrections [4] which can then be neglected for our purposes. In the (generally unrealistic) assumption of diagonal charged leptons, the mixing angles would be given by $\theta_{12} = \pi/4$ (i.e. maximal solar angle), $\tan \theta_{23} = x$ (i.e. generically large atmospheric angle) and $\theta_{13} = 0$. As the observed mixing angles show the opposite pattern, with a definitely non maximal solar angle and an atmospheric angle compatible with being maximal [5], a substantial correction to the mixing angles is required from this starting approximation. The physical value of $r = \Delta m_{sol}^2 / \Delta m_{atm}^2 \sim 1/30$ [5], which is zero in the symmetric limit, must also be obtained from the corrective terms. A main problem is that simple forms of $U(1)_F$ symmetry breaking tend to produce too small a deviation of the solar angle from its maximal value [6–8]. However, the correction to bring the solar angle down from its maximal value can be obtained as an effect of the charged lepton matrix diagonalization [9]. This possibility, studied in detail in refs. [10–12], in abstract terms is not excluded but is strongly constrained by the observed smallness of θ_{13} , as the amount of deviation from a maximal solar angle is of order θ_{13} . This mechanism can only work if the value of θ_{13} is very close to its present upper bound. Within the $U(1)_F$ framework the question of which symmetry breaking mechanism is capable of leading to a suitable charged lepton mass matrix for this purpose is left completely unanswered.

Recently Grimus and Lavoura (GL) [13] suggested that, in a see-saw realization of the model, a large breaking of $U(1)_F$ could be present in M_{RR} , the right-handed neutrino Majorana matrix. A crucial point is that a large symmetry breaking in M_{RR} cannot propagate via radiative corrections to other sectors of the lagrangian given that ν_R has no gauge interactions. One could possibly reproduce the observed values of the solar and atmospheric mixing angles together with a small value of θ_{13} (which could even be zero). We present here a detailed discussion of models based on this idea. With respect to GL there are important additions and differences: we have three right-handed neutrinos, we explicitly discuss the charged lepton sector (in particular, we aim at reproducing the mass hierarchy in a natural way) and we complete the model with additional structure at the non leading level. The large breaking of $U(1)_F$ from M_{RR} produces the required large correction to fix θ_{12} with θ_{13} still vanishing at this level. An additional, more conventional, form of $U(1)_F$ symmetry breaking in terms of vev's of a flavon field produces a pattern of non leading corrections. These terms provide the correct hierarchy for the charged lepton masses and, through the effect of the charged lepton diagonalization, shift the θ_{13} value away from zero by a small amount, of the order of a power of charged lepton mass ratios. In this way a considerable freedom in the possible range of θ_{13} is made compatible with the observed values of the solar and atmospheric mixing angles. However, a moderate fine tuning is necessary to reproduce the observed smallness of r that normally should be $\sim o(1)$ in this kind of models.

2. Models with Broken $L_e - L_\mu - L_\tau$

In the leptonic sector, we adopt the following classification under $U(1)_F$:

$$\begin{aligned}
 l_i &\sim L_e - L_\mu - L_\tau \sim (1, -1, -1) \\
 l_{Ri} &\sim (Q_e, Q_\mu, -1) \\
 \nu_{Ri} &\sim (-Q_R, Q_R, 0).
 \end{aligned}
 \tag{2.1}$$

The Higgs doublet H is taken to carry $Q_H = 0$. In presence of only one Higgs doublet this does not imply a loss of generality. Note that, for right handed charged leptons, the $U(1)_F$ charge assignment does not coincide with $L_e - L_\mu - L_\tau$ (the latter would lead to no structure for charged lepton masses). The $U(1)_F$ symmetry is broken by the vev of a complex field θ of charge $Q_\theta = 1$ (and θ^\dagger has, of course, charge -1 , so that one has the same suppression for either sign of the charge mismatch). The suppression factor for a unit charge mismatch is $\lambda = \langle \theta \rangle / \Lambda$, with Λ being the cut-off. The resulting charged lepton mass matrix is of the form (with coefficients of $o(1)$ at each entry always understood as in all $U(1)_F$ models):

$$m^l \sim \bar{l}_R \sim m_\tau \begin{pmatrix} \lambda^{-1+Q_e} & \lambda^{-1+Q_\mu} & \lambda^2 \\ \lambda^{1+Q_e} & \lambda^{1+Q_\mu} & 1 \\ \lambda^{1+Q_e} & \lambda^{1+Q_\mu} & 1 \end{pmatrix} \sim m_\tau \begin{pmatrix} \xi' & \xi \epsilon & \epsilon \\ \xi' \epsilon & \xi & 1 \\ \xi' \epsilon & \xi & 1 \end{pmatrix}.
 \tag{2.2}$$

where we have set

$$\epsilon \sim \lambda^2, \quad \xi \sim \lambda^{|1+Q_\mu|} \sim \frac{m_\mu}{m_\tau} \sim 6 \cdot 10^{-2}, \quad \xi' \sim \lambda^{-1+Q_e} \sim \frac{m_e}{m_\tau} \sim 3 \cdot 10^{-4}.
 \tag{2.3}$$

λ	Q_e	Q_μ	Q_τ
$0.25 \sim \xi^{\frac{1}{2}}$	7	-3	-1
$0.15 \sim \xi^{\frac{2}{3}}$	$\frac{11}{2}$	$-\frac{5}{2}$	-1
$0.06 \sim \xi$	4	-2	-1
$4 \times 10^{-3} \sim \xi^2$	$\frac{5}{2}$	$-\frac{3}{2}$	-1
$2 \times 10^{-4} \sim \xi^3$	2	$-\frac{4}{3}$	-1

Table 1: A set of charges and corresponding values of λ that can accommodate the observed charged lepton masses. In the following, we will argue that, in this class of models, we expect $\theta_{13} \leq o(\lambda)$, so that we have restricted us to values of λ below the upper bound for θ_{13} .

Note that the charged lepton mass ratios are almost insensitive to the renormalization group scale from the electroweak up to the GUT energy [14]. In eq. (2.2) we have taken $Q_\tau = -1$ in order to make $m_\tau \sim o(1)$. The values of λ , Q_e and Q_μ are to be chosen in such a way as to reproduce the correct pattern of masses. In particular the signs of Q_e and Q_μ are fixed in order to obtain a charged lepton mass matrix with the structure shown in eq. (2.5). Also, we will see in the following that θ_{13} will end up being of order λ , so that we take $\lambda < 0.25$. Then a set of possible values is shown in table 1.

The charged lepton mass matrix, eq. (2.2), can be related to its diagonal form by multiplication on the left by:

$$U_l = R_{23}(\theta_l)R_{13}(\epsilon)R_{12}(\epsilon) \tag{2.4}$$

where $R_{ij}(\theta)$ is a rotation in the ij plane of angle θ , θ_l is an angle of $o(1)$ and by ϵ we indicate an angle of order $\epsilon \sim \lambda^2$ [8]. The relation is fixed by:

$$\begin{pmatrix} \xi' & \xi\epsilon & \epsilon \\ \xi'\epsilon & \xi & 1 \\ \xi'\epsilon & \xi & 1 \end{pmatrix} = U_l \begin{pmatrix} \xi' & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.5}$$

The matrix U_l contributes to the neutrino mixing matrix via the relation $U_{PNMS} = U_l^\dagger U_\nu$. The resulting effect is a shift of θ_{23} by a large angle proportional to θ_l , while θ_{12} and θ_{13} are corrected by small terms of $o(\epsilon \sim \lambda^2)$. We shall see that larger contributions of order λ to θ_{13} may arise from the neutrino sector in the present model. We clearly see that, in the framework of $U(1)_F$ broken by vev's, θ_{13} is normally forced to be small by the observed values of the charged lepton mass ratios. This is because $\theta_{13} \sim o(\lambda^2)$ and λ must be small in order for ξ and ξ' to reproduce the observed mass ratios.

We now consider the neutrino mass sector. The value of the charge Q_R in eq. (2.1) must be specified. Only for an absolute value $|Q_R| = 1$ we obtain that some Dirac neutrino mass matrix elements are unsuppressed, and the sign choice actually leads to equivalent models for all practical purposes (after some parameter reshuffling and neglecting second order effects in the $U(1)_F$ breaking). Thus we take $Q_R = 1$ in the following. The neutrino Dirac matrix is then of the form:

$$m_\nu^D \sim \bar{\nu}_R l \sim m \begin{pmatrix} y_{11}\lambda^2 & a & b \\ d & y_{22}\lambda^2 & y_{23}\lambda^2 \\ y_{31}\lambda & y_{32}\lambda & y_{33}\lambda \end{pmatrix}. \tag{2.6}$$

Here we have explicitly indicated the coefficients y_{ij} , a , b and d , all of $o(1)$. Similarly for the neutrino right-handed Majorana matrix we should write:

$$m_{RR}^{-1} \sim \frac{1}{M} \begin{pmatrix} x_{11}\lambda^2 & W & x_{13}\lambda \\ W & x_{22}\lambda^2 & x_{23}\lambda \\ x_{13}\lambda & x_{23}\lambda & Z \end{pmatrix}. \quad (2.7)$$

Note that, in this case, the orders of magnitude of the various entries would be the same in m_{RR}^{-1} and m_{RR} . But, as explained in the introduction, we assume, following GL, that in m_{RR} a different source of $U(1)_F$ breaking could be operating in addition to that induced by the flavon vev's, in the form of explicit soft mass terms. We thus write down a generic form for m_{RR}^{-1} :

$$m_{RR}^{-1} \sim \frac{1}{M} \begin{pmatrix} A & W & B \\ W & C & D \\ B & D & Z \end{pmatrix}. \quad (2.8)$$

In the notation adopted W and Z are the terms that are already present in the symmetric limit, while A , B , C and D are symmetry breaking terms. While we keep in mind, for use in the following, that in the case of $U(1)_F$ broken by flavon vev's, $|A|, |C| \sim \lambda^2$ and $|B|, |D| \sim \lambda$, we make no statements about the orders of magnitude of each entry at this stage. From the see-saw formula we obtain:

$$\begin{aligned} m_\nu &= m_\nu^{DT} m_{RR}^{-1} m_\nu^D \sim m_{\nu 0} + \lambda m_{\nu 1} + \dots \sim \\ &\sim \frac{m^2}{M} \begin{pmatrix} d^2 C & adW & bdW \\ adW & a^2 A & abA \\ bdW & abA & b^2 A \end{pmatrix} + \\ &+ \lambda \frac{m^2}{M} \begin{pmatrix} 2y_{31}dD & y_{31}aB + y_{32}dD & y_{31}bB + y_{33}dD \\ y_{31}aB + y_{32}dD & 2y_{32}aB & y_{32}bB + y_{33}aB \\ y_{31}bB + y_{33}dD & y_{32}bB + y_{33}aB & 2y_{33}bB \end{pmatrix} + o(\lambda^2). \end{aligned} \quad (2.9)$$

We note that Z does not appear at all to $o(\lambda)$, while A , C and W only appear in $m_{\nu 0}$ and B and D only appear in $m_{\nu 1}$. Note that, in the case of $U(1)_F$ broken by flavon vev's, $|A|, |C| \sim \lambda^2$ and $|B|, |D| \sim \lambda$, so that all entries in $m_{\nu 0} + \lambda m_{\nu 1}$ are either of $o(1)$ (the W terms) or of $o(\lambda^2)$. In this case the pattern for the neutrino mass matrix would be exactly the same even for no right-handed neutrinos and no see-saw, with $m_\nu \sim l^T l$ obtained directly from the $L_e - L_\mu - L_\tau$ charges of the lepton doublets l . Instead, if the dominant breaking of $U(1)_F$ is through mass terms in m_{RR}^{-1} , the see-saw generation of masses is clearly essential and the resulting breaking can be larger, for example by terms of $o(1)$ in $m_{\nu 0}$ (the A and C terms), and by terms of $o(\lambda)$ in $\lambda m_{\nu 1}$ (the B and D terms).

We can analyse the results at various levels of symmetry breaking. For exact $U(1)_F$ we can set $\lambda = 0$ and keep only the terms in W , a , b and d . At this level we get the well known predictions of exact $L_e - L_\mu - L_\tau$:

$$m_1 = -m_2; \quad m_3 = 0; \quad \theta_{13} = 0; \quad \theta_{12} = \frac{\pi}{4}; \quad \tan \theta_{23} = \frac{b}{a}. \quad (2.10)$$

At the next level, we only take $m_{\nu 0}$ in eq. (2.9), i.e. we drop all λ terms and only keep the symmetry breaking terms from A and C . Note that in general there are three phases left in the physical light neutrino mass matrix. However, since for $m_{\nu 0}$ both m_3 and θ_{13} vanish, only one single phase is left. We can choose, for example, to take \bar{W} and $\bar{A} - \bar{C}$ real and write $\bar{A} + \bar{C} = -|\bar{A} + \bar{C}|e^{i\delta}$. We then obtain:

$$\begin{aligned} m_3 &= 0; \quad m_1 + m_2 = \bar{C} + \bar{A}; \quad m_1 - m_2 = \sqrt{(\bar{A} - \bar{C})^2 + \bar{W}^2}; \\ \theta_{13} &= 0; \quad \tan \theta_{23} = \left| \frac{b}{a} \right|; \quad \tan^2 2\theta_{12} \sim \frac{\bar{W}^2}{(\bar{A} - \bar{C})^2} \end{aligned} \quad (2.11)$$

where we have set:

$$\bar{A} = \frac{m^2}{M} A (a^2 + b^2); \quad \bar{C} = \frac{m^2}{M} C d^2; \quad \bar{W}^2 = \frac{m^2}{M} 4W^2(a^2 + b^2)d^2. \quad (2.12)$$

We stress that, at this level, all results are completely equivalent to those obtained in the model of GL where only two right-handed neutrinos are assumed. In fact, the presence of a third ν_R does not introduce additional parameters as only A , C and W from the 1-2 submatrix of m_{RR}^{-1} appear in $m_{\nu 0}$. At this stage we see that in order to obtain a realistic value for $\tan^2 2\theta_{12}$ we need the symmetry breaking parameter $|\bar{A} - \bar{C}|$ to be not too small with respect to the symmetric term \bar{W} . Experimentally, at $2\text{-}\sigma$, $\sin^2 \theta_{12} \sim 0.314 + 0.06 - 0.05$ [5] so that $\tan^2 2\theta_{12} = t^2 \sim 6$, and we obtain:

$$\frac{|\bar{A} - \bar{C}|}{|\bar{W}|} \sim 0.40. \quad (2.13)$$

On the other hand, r is given by:

$$r = (|m_2|^2 - |m_1|^2)/|m_2|^2 = \frac{4|\bar{A} + \bar{C}| \cos \delta \sqrt{(\bar{A} - \bar{C})^2 + \bar{W}^2}}{|\bar{A} + \bar{C} + \sqrt{(\bar{A} - \bar{C})^2 + \bar{W}^2}|^2} \sim \frac{4\eta \cos \delta \sqrt{1 + t^2}}{|-\eta e^{i\delta} + \sqrt{1 + t^2}|^2}. \quad (2.14)$$

Note that we take $\cos \delta > 0$ in order that $|m_2|^2 - |m_1|^2 > 0$ in agreement with the usual definition of $m_{1,2}$. The numerical values for η and $\cos \delta$ are constrained by the observed value of r , $r \sim 1/30$ [5]. For example, if η is small one finds approximately:

$$\eta \cos \delta = \left| \frac{\bar{A} + \bar{C}}{\bar{A} - \bar{C}} \right| \cos \delta \sim 0.02. \quad (2.15)$$

We have discarded the possibility of very large η because $m_1 + m_2 = \bar{A} + \bar{C}$ is zero in the exact symmetry limit. We see that a moderate amount of fine tuning is needed to fix the observed value of r which would naturally be expected to be of $\sim o(1)$ in this model.

We now consider the effects at order λ induced by the term $\lambda m_{\nu 1}$ in eq.(2.9). First of all, in general all quantities which were vanishing for $\lambda = 0$ are now different from zero. In particular, θ_{13} receives a contribution at order λ given by:

$$|\tan \theta_{13}| \sim \left| \lambda \left[\frac{B}{W} - \frac{A}{W} \frac{D}{W} \right] \frac{y_{33}a - y_{32}b}{d\sqrt{a^2 + b^2}} \right|. \quad (2.16)$$

As we have already mentioned, we expect B and D in the range $o(\lambda) \leq |B|, |D| \leq o(1)$, the lower limit corresponding to the flavon vev breaking of $U(1)_F$ and the upper limit to the size of the soft breaking terms A and C . Thus $\tan \theta_{13}$ acquires a non vanishing value $o(\lambda^2) \leq |\tan \theta_{13}| \leq o(\lambda)$. We recall that a contribution of $o(\lambda^2)$ is in any case expected from the diagonalization of the charged lepton matrix. Going back to Table 1 we see that all measurable values for θ_{13} can be obtained with a suitable choice of the $U(1)_F$ charges as well as values below the experimentally accessible range. Thus, in this class of models, there is no difficulty in accomodating a large deviation of the solar angle θ_{12} from the maximal value together with a size of θ_{13} much below the present experimental bound. Another quantity which vanishes for $\lambda = 0$ is m_3 . It is easy to see that m_3 remains zero at $o(\lambda)$ and only receives a contribution at next order: $m_3 \sim o(\lambda^2)$. In fact, the neutrino Dirac mass matrix m_ν^D in eq.(2.6) has a determinant of $o(\lambda)$ so that m_ν obtained from the see-saw formula in eq.(2.9) has a determinant of $o(\lambda^2)$. Thus the corrected mass spectrum is still of the inverse hierarchy type. The quantity $r = \Delta m_{12}^2 / \Delta m_{23}^2$ is corrected by terms of $o(\lambda)f_r(B, D)$, where $f_r(B, D) \sim o(\lambda) - o(1)$. Given that the observed value of r is small, $r \sim 1/30$, if $f_r(B, D) \sim o(1)$ then the largest values of λ shown in Table 1 would be disfavoured, and the case $\theta_{13} \sim r$ or smaller would be indicated, barring cancellations or additional fine tuning. All other quantities like θ_{12}, θ_{23} etc receive corrections of $o(\lambda)f(B, D)$, which however are not important as the leading terms are $o(1)$.

3. Conclusion

In this note we have reconsidered models of neutrino masses with inverse hierarchy based on a broken $U(1)_F$ symmetry with charge $Q_l = L_e - L_\mu - L_\tau$. These models can reproduce many of the observed features of the charged lepton spectrum and of the neutrino mixing angles, but a completely natural model is difficult to realize. The problem is that, in the limit of unbroken $U(1)_F$, $1 - \tan \theta_{12}$, r and θ_{13} are all zero. This is also true for the charged lepton mass ratios m_e/m_τ and m_μ/m_τ for our choice of right-handed charges (motivated by the observed smallness of these ratios). Experimentally, however, $1 - \tan \theta_{12}$ turns out to be large. The difficulty is to have a large symmetry breaking correction for this particular observable while keeping all other deviations from the symmetric limit small. In fact, if the symmetry breaking occurs through a number of flavon vev's, the symmetry breaking order parameter λ must be small to fit the charged lepton mass ratios as well as r and θ_{13} . Then $1 - \tan \theta_{12}$ in general would also be small. The idea of disentangling θ_{12} from the maximal value as an effect of the charged lepton diagonalization involves some amount of stretching, because it needs θ_{13} to be very near its present upper bound, and, also, the question of why they turn out to be rather large is usually not addressed. The version of the model, studied in detail here, where, to the usual breaking of $U(1)_F$ by vev's, one adds a large source of symmetry breaking in the m_{RR} mass sector is remarkable, because it allows a natural separation of $1 - \tan \theta_{12}$, which indeed becomes large, from the charged lepton mass ratios and θ_{13} , which are naturally predicted to be small. The only remaining imperfection is that, in this case, $1 - \tan \theta_{12}$ and r should be of the same order, in particular both of $o(1)$ while they differ by a factor of about 10. This moderate fine tuning is the

price to pay for an otherwise natural model with a simple structure. Finally, in this model, for m_{ee} , the parameter measured in neutrinoless double beta decay, one obtains at 2σ the range $0.012 \leq |m_{ee}| \leq 0.054$, that could be reached by the next generation of experiments.

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